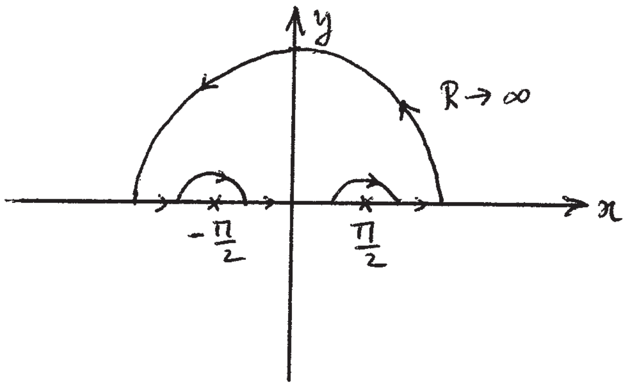
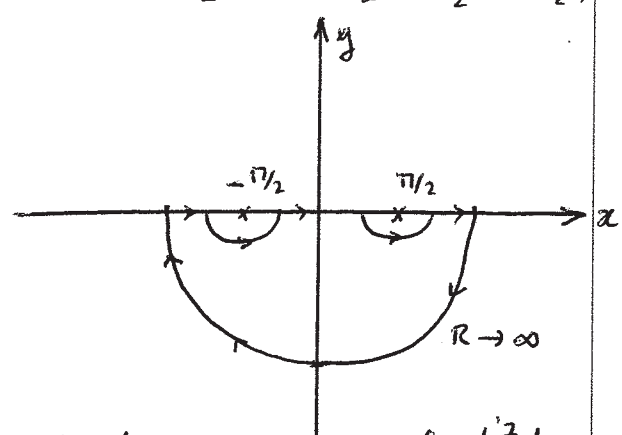
Problem 27) $$\int_{-\infty}^{\infty} \frac{\cos x}{\pi^2 - 4x^2}\, dx = -\frac{1}{8}\int_{-\infty}^{\infty} \frac{e^{ix}\, dx}{(x-\frac{\pi}{2})(x+\frac{\pi}{2})} - \frac{1}{8}\int_{-\infty}^{\infty} \frac{e^{-ix}\, dx}{(x-\frac{\pi}{2})(x+\frac{\pi}{2})}$$





The first integral is taken in the upper half plane, where $\int \frac{e^{iz}dz}{z^2 - (\frac{\pi}{2})^2} \to 0$ as $R \to \infty$ (Jordan's Lemma). The second integral is taken in the lower half-plane for the same reason.

The poles at $z = \pm\pi/2$ are simple poles, but they contribute only half of the corresponding residue to the integral, because they are located <u>on</u> the contour (as opposed to <u>inside</u> the contour).

1st integral, residue at $z_1 = \frac{\pi}{2}$: $\left.\frac{e^{iz_1}}{z_1 - z_2}\right|_{z_2 = -\frac{\pi}{2}} = \frac{e^{i\pi/2}}{\pi} = \frac{i}{\pi}$. ✓

1st integral, residue at $z_2 = -\frac{\pi}{2}$: $\frac{e^{iz_2}}{z_2 - z_1} = \frac{e^{-i\pi/2}}{-\pi} = \frac{-i}{-\pi} = \frac{i}{\pi}$. ✓

2nd integral, residue at $z_1 = \frac{\pi}{2}$: $-\frac{e^{-iz_1}}{z_1 - z_2} = -\frac{e^{-i\pi/2}}{\pi} = \frac{i}{\pi}$. ✓

direction of travel around small semi-circle reversed

2nd integral, residue at $z_2 = -\frac{\pi}{2}$: $-\frac{e^{-iz_2}}{z_2 - z_1} = -\frac{e^{i\pi/2}}{-\pi} = \frac{i}{\pi}$. ✓

Therefore $$\int_{-\infty}^{\infty} \frac{\cos x}{\pi^2 - 4x^2}\, dx = -\frac{1}{8}(\pi i)\left(\frac{i}{\pi} + \frac{i}{\pi}\right) - \frac{1}{8}(\pi i)\left(\frac{i}{\pi} + \frac{i}{\pi}\right) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}.$$ ✓

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